Reg. No. : ....

Code No:6834 Sub. Code: PMAM14

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021

FIRST SEMESTER

MATHEMATICS – CORE

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time: Three hours Maximum : 75 marks

PART A — (10 X 1 = 10 marks)

Answer ALL questions, Choose the correct answer

1. If $y_1(x)$ and $y_2(x)$ are two solutions $y'' + P(x)y' + Q(x)y = 0$, then $W =$

(a) ce^{-SPdx}

(b) ce^{-SQdx}

(c) ce^{+SPdx}

(d) ce^{+SQdx}

2. Two linearly independent solutions of $y'' - y = 0$ are

(a) 1 and e^x (b) 1 and e^{-x}

(c) e^x and e^{-x} (d) None

3. The equation $y' = y$ has a power series solution with radius of convergence R where

(a) $R > 0$ (b) $R < 0$

(c) $R = 0$ (d) None

4. The particular solution of $(1-x^2)\, y'' - 2xy' + p(p+1)y = 0$ where p is a constant is known as

(a) Bessels equation (b) Legendre equation

(c) Hermite equations (d) AIRY's equation

5. The singular point of $x^2y'' + 2xy' - 2y = 0$ is

(a) 1 (b) ∞

(c) 0 (d) –1

6. The Rodriques formula $p_n(x) =$ is

(a) $\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[x^2-1\right]^n$ (b) $\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[x^2+1\right]^n$

(c) $\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[x^2-1\right]^{-n}$ (d) $\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[x^2+1\right]^{-n}$

7. The value of $J_{y2}(x)$ is

(a) $\sqrt{\frac{2}{\pi x}}\sin x$

(b) $\sqrt{\frac{2}{\pi x}}\cos x$

(c) $\sqrt{\frac{2}{\pi x}}\tan x$

(d) None

8. The famous $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ whose sum was discovered by

(a) Newton

(b) Euler

(c) Bessel

(d) Legendre

9. The vanishing or non vanishing of the Wronskian $w(t)$ of two solutions ———————— on the circle of t.

(a) Depends

(b) Does not depend

(c) Either (a) or (b)

(d) None

10. By solving the linear systems using auxillary equation. If m_1 and m_2 all distinct real numbers then the roots are ————————.

(a) Same

(b) Distinct

(c) Trivial

(d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, by choosing either (a) or (b).

11. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Then prove that $W = W(Y_1, Y_2)$ is either identically zero or never zero on $[a, b]$.

Or

(b) The equation $y'' - y = 0$ has a solution $y_1 = e^x$. Find y_2 and the general solution.

12. (a) Find a power series solution of the equation $y' = y$.

Or

(b) Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ interms of power series.

13. (a) Find the indicial equation and its roots the equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.

Or

(b) Find the first three terms of the Legendre series $f(x) = e^x$.

14. (a) Prove that : $\frac{d}{dx}[J_0(x)] = -J_1(x)$.

Or

(b) Show that : $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

15. (a) Prove that if the solutions of the homogeneous system are linearly independent on *[a, b]*. Then the system is the general solution of homogeneous solution of this interval.

Or

(b) Find the general solution of $\frac{dx}{dt} = 2x$, $\frac{dy}{dt} = 3y$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, by choosing either (a) or (b).

16. (a) Show that $y = c_1 e^x + e_2 e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval and find the particular solution for which $y(0) = 1$ and $y'(0) = 1$.

Or

(b) If y_1 is a non zero solution of the equation $y'' + P(x)y' + Q(x)y = 0$ and $y_2 = vy_1$, where v is given by the formula $v = \int \frac{1}{y_1^2} e^{-\int p dx} dx$ is the second solution. Show by computing the Wronkian that y_1 and y_2 are linearly independent.

17. (a) Show that $y = (1+x)^p$ is a power series solution of the equation $(1+x)y' = p_y$, $y(0) = 1$.

Or

(b) Find the general solution of $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$, where p is a constant.

18. (a) Show that the indicial equation of the equation $x^2y'' + xy' + x^2y = 0$ has only one root and prove that $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$.

Or

(b) Show that :

$$\int_{-1}^{1} p_m(x) p_n(x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}.$$

19. (a) Prove that $J_{-m}(x) = (-1)^m J_m(x)$.

Or

(b) Prove that

$$\int_0^1 x\, J_p(\lambda_m x) J_p(\lambda_n x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} J_{p+1}(\lambda_n)^2 & \text{if } m = n \end{cases}$$

20. (a) Show that the Wronskian of the two solutions in distinct complex roots is given by $w(t) = (A_1 B_2 - A_2 B_1) e^{2at}$ and prove that $A_1 B_2 - A_2 B_1 \neq 0$.

Or

(b) Find the general solution of

$$\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y.$$
